NOTATION

U is the longitudinal component of the mean velocity; u_1 is the longitudinal component of the velocity fluctuations; M is the size of the grating cell; d is the diameter of the grating rods, Re_λ is the turbulent Reynolds number; $E_1(f)$ is the energy of the longitudinal velocity fluctuations; ε_1 is dissipation of the longitudinal velocity fluctuations; ν is the kinematic viscosity; x_1 is the longitudinal coordinate; f_S is the sampling frequency, and f_C is the cutoff frequency.

LITERATURE CITED

1. B. J. S. Barnard and R. H. Y. Sellin, Nature (London), 222, No. 5199, 1160-1164 (1969).
2. W. D. McComb, J. Allan, and C. A. Greated, Phys. Fluids, 20, NO. 6, 873-879 (1977).
3. C. A. Friehe and W. H. Schwarz, J. Fluid Mech., 44, No. 1, 173-193 (1971).
4. J. S. Chung and W. P. Graebel, Phys. Fluids, 15, No. 4, 546-554 (1972).
5. S. E. Logan, AIAA Jnl, 10, No. 7, 962-964 (1972).
6. M. M. Reischmen and W. G. Tiederman, J. Fluid Mech., 70, No. 2, 369-392 (1975).
7. R. I. Soloukhin, B. A. Kolovandin, B. I. Puris, et al., Turbulent Transfer Processes [in Russian], 3-15, Minsk (1985).
8. J. O. Hintze, Turbulence [Russian translation], Moscow (1963).
9. M. S. Uberoi and S. Wellis, Phys. Fluids, 10, No. 6, 1216-1224 (1967).
10. W. D. McComband and L. H. Rabie, AIChE, 28, No. 4, 558-565 (1982).

INFLUENCE OF PARTICLES ON THE INITIAL STAGE OF HOMOGENEOUS TURBULENCE DEGENERATION

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The influence of particles on the degeneration of homogeneous turbulence at high Reynolds numbers is analyzed.

On the basis of the solution of a system of equations for two-point moments of gas and particle velocity fluctuations without taking account of the triple correlations, the influence of the disperse phase on the final stage of homogeneous isotropic turbulence degeneration is investigated in [1] for moderate Reynolds numbers; it is obtained here that the presence of particles magnifies the damping of the turbulent fluctuations. It is noted in [2] that particles in a jet result in two oppositely directed effects: on one hand, a reduction in the fluctuation intensity of the lifting stream velocity occurs because of additional turbulent energy dissipation, and on the other, growth of turbulent energy generation occurs because of the increase in the average gas velocity gradient. Attenuation of the turbulent fluctuation intensity is established for near-wall flows in [3, 4] because of the additional dissipation in the presence of coarse particles in the stream, and conversely, a rise in the fluctuation intensity because of additional turbulence generation due to the average flow in the case of fine particles present in the stream. Therefore, depending on the inertia characterized by the ratio between the relaxation time to the time scale of the turbulence, the particles can contribute to both laminarization and turbulization of the stream. Such a regularity of the particle influence on turbulence is inherent to different kinds of flows. The nature of the particle influence on the turbulent fluctuation intensity is illustrated in this paper in an example of the problem of homogeneous turbulence degeneration for high Reynolds numbers.

The flow of a gas stream with solid particles ($\rho_2 >> 1$) for a moderate volume concentration of the disperse phase ($\varphi \ll 1$) is considered. The equations of motion for the gas and particles are written in a Stokes approximation in the form

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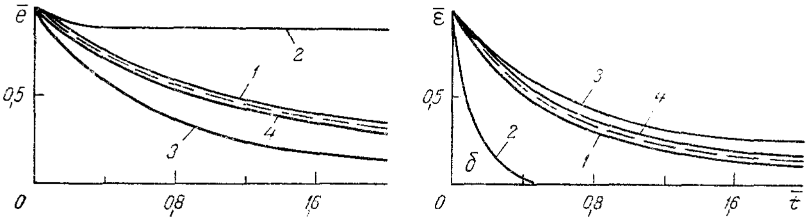

Fig. 1. Change in turbulent energy (a) and dissipation (b) in time: 1) Ω = 0.01; 2) 0.1; 3) 1; 4) 10.

$$\frac{\partial u_i}{\partial t}+u_k\frac{\partial u_i}{\partial x_k}=-\frac{1}{\rho_1}\frac{\partial P}{\partial x_i}+\nu\frac{\partial^2 u_i}{\partial x_k\partial x_k}-\frac{\rho_2\varphi}{\rho_1\tau}(u_i-v_i), \tag{1}$$

$$\frac{dv_i}{dt}=\frac{u_i-v_i}{\tau}. \tag{2}$$

The equations for the second one-point moments of the lifting phase velocity fluctuations $\langle u_i'u_j'\rangle$ without taking account of the particle concentration fluctuations in conformity with (1) have the form

$$\begin{aligned}\frac{\partial\langle u_i'u_j'\rangle}{\partial t}+U_k\frac{\partial\langle u_i'u_j'\rangle}{\partial x_k}&=-\langle u_i'u_k'\rangle\frac{\partial U_j}{\partial x_k}-\\ &-\langle u_j'u_k'\rangle\frac{\partial U_i}{\partial x_k}-\frac{\partial\langle u_i'u_j'u_k'\rangle}{\partial x_k}-\\ &-\frac{1}{\rho_1}\left(\left\langle u_i'\frac{\partial P'}{\partial x_j}\right\rangle+\left\langle u_j'\frac{\partial P'}{\partial x_i}\right\rangle\right)+\nu\frac{\partial^2\langle u_i'u_j'\rangle}{\partial x_k\partial x_k}-\\ &-2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_j'}{\partial x_k}\right\rangle-\frac{\rho_2\varphi}{\rho_1\tau}(2\langle u_i'u_j'\rangle-\langle v_i'u_j'\rangle+\langle v_j'u_i'\rangle).\end{aligned} \tag{3}$$

The particle influence on the turbulent stress is described by the last component in (3). The mixed correlation moment of the solid and gas phase velocity fluctuations in this component is expressed in terms of the second moment of the gas phase velocity fluctuations by using the relationship

$$\langle v_i'u_j'\rangle=\frac{1}{\tau}\int_0^\infty F(s)\exp(-s/\tau)\,ds\,\langle u_i'u_j'\rangle, \tag{4}$$

obtained from (2). Here $F(s)=\langle u_i'(t)u_j'(t+s)\rangle/\langle u_i'(t)u_j'(t)\rangle$ is the two-time correlation function of the gas stream velocity fluctuations. As in [3, 4], we approximate F(s) by a step function

$$F(s)=\begin{cases}1 & \text{for}\quad 0\leqslant s\leqslant T,\\ 0 & \text{for}\quad s>T,\end{cases} \tag{5}$$

where T is the time integral turbulence scale determining the life time of the energetic moles. Taking (5) into account, (4) takes the form

$$\langle v_i'u_j'\rangle=[1-\exp(-1/\Omega)]\langle u_i'u_j'\rangle, \tag{6}$$

where $\Omega=\tau/T$ is a parameter characterizing the particle inertia from the viewpoint of their involvement in the lifting stream fluctuating motion.

The equation for the scalar dissipation of the fluctuation energy $\varepsilon=\nu(\partial u_i'/\partial x_k\cdot\partial u_i'/\partial x_k)$ is obtained from (1)

$$\frac{\partial\varepsilon}{\partial t}+U_k\frac{\partial\varepsilon}{\partial x_k}=-2\nu\left(\left\langle\frac{\partial u_i'}{\partial x_n}\frac{\partial u_k'}{\partial x_n}\right\rangle+\left\langle\frac{\partial u_n'}{\partial x_i}\frac{\partial u_n'}{\partial x_k}\right\rangle\right)\frac{\partial U_i}{\partial x_k}-$$

$$-2\nu\left\langle u_k'\frac{\partial u_i'}{\partial x_n}\right\rangle\frac{\partial^2 U_i}{\partial x_n\partial x_k}-2\nu\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_n}\frac{\partial u_k'}{\partial x_n}\right\rangle-$$

$$-\nu\frac{\partial}{\partial x_k}\left\langle u_k'\left(\frac{\partial^2 u_i'}{\partial x_n^2}\right)^2\right\rangle-\frac{2\nu}{\rho_1}\frac{\partial}{\partial x_k}\left\langle\frac{\partial P'}{\partial x_n}\frac{\partial u_k'}{\partial x_n}\right\rangle-$$

$$-2\nu^2\left\langle\left(\frac{\partial^2 u_i'}{\partial x_n\partial x_k}\right)^2\right\rangle+\nu\frac{\partial^2\varepsilon}{\partial x_k\partial x_k}- \tag{7}$$

$$-\frac{2\nu\rho_2\varphi}{\rho_1\tau}\left(\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle-\left\langle\frac{\partial v_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle\right)-$$

$$-\frac{2\nu\rho_2}{\rho_1\tau}\left(\left\langle u_i'\frac{\partial u_i'}{\partial x_k}\right\rangle-\left\langle v_i'\frac{\partial u_i'}{\partial x_k}\right\rangle\right)\frac{\partial\varphi}{\partial x_k}.$$

The particle influence on fluctuation energy dissipation is determined by the last two terms in (7), where only the first of them turns out to be essential at high Reynolds numbers. Analogously to (4), an expression is obtained for the mixed correlation moment between the derivatives of the solid and gas phase velocities

$$\left\langle\frac{\partial v_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle=\frac{1}{\tau}\int_0^\infty F_\varepsilon(s)\exp\left(-\frac{s}{\tau}\right)ds\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle, \tag{8}$$

where

$$F_\varepsilon(s)=\left\langle\frac{\partial u_i'(t)}{\partial x_k}\frac{\partial u_i'(t+s)}{\partial x_k}\right\rangle\Big/\left\langle\frac{\partial u_i'(t)}{\partial x_k}\frac{\partial u_i'(t)}{\partial x_k}\right\rangle$$

is the autocorrelation function of the derivative of the gas stream velocity fluctuations. The time interval in which damping of the function $F_\varepsilon(S)$ occurs in contrast to $F(s)$ is not determined by the macroscale T but by the microscale. For high Reynolds numbers the relationships $T/T_\varepsilon\sim\mathrm{Re}^{1/2}$ holds. Analogously to (5) we approximate $F_\varepsilon(s)$ by a step function by replacing the quantity T by T_ε. Then (8) takes the form

$$\left\langle\frac{\partial v_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle=\left[1-\exp\left(-\frac{\alpha}{\Omega}\right)\right]\left\langle\frac{\partial u_i'}{\partial x_k}\frac{\partial u_i'}{\partial x_k}\right\rangle, \tag{9}$$

where $\alpha = T_\varepsilon/T$ is a parameter characterizing the ratio between the micro and macroscales.

In the case of homogeneous turbulence at high Reynolds numbers (3) and (7) simplify substantially. Let us write the equations for turbulent energy and turbulent energy dissipation in dimensionless form with (6) and (9) taken into account

$$\frac{d\bar e}{d\bar t}=-\bar\varepsilon-\frac{2\exp(-1/\Omega)\Phi}{\Omega}\bar e, \tag{10}$$

$$\frac{d\bar\varepsilon}{d\bar t}=-2\frac{\bar\varepsilon^2}{\bar e}-\frac{2\exp(-\alpha/\Omega)\Phi}{\Omega}\bar\varepsilon. \tag{11}$$

Only terms taking account of viscous dissipation and additional dissipation due to interphasal fluctuating slip remain in the right sides of (10) and (11). In the absence of particles ($\Phi = 0$) known relationships for the damping of turbulence at high Reynolds numbers [5, 6] follow from (10) and (11)

$$\bar e=(1+\bar t)^{-1},\quad \bar\varepsilon=(1+\bar t)^{-2}. \tag{12}$$

Let us find the solution of (10) and (11) for fixed values of the parameters Ω and α, i.e., by setting $\Omega = \tau/T_0$, $\alpha = T_{\varepsilon 0}/T_0$. In this case the solution of (10) and (11) will be

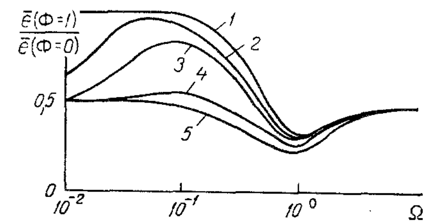

Fig. 2. Influence of particle inertia on the turbulent energy: 1) $\alpha = 0$; 2) 0.05; 3) 0.1; 4) 0.5; 5) 1.

$$\bar{e} = \frac{\exp[-2\exp(-1/\Omega)\,\Phi\,\bar{t}/\Omega]}{1 + \dfrac{1-\exp[-2(\exp(-\alpha/\Omega)-\exp(-1/\Omega))\,\Phi\bar{t}/\Omega]}{2(\exp(-\alpha/\Omega)-\exp(-1/\Omega))\,\Phi/\Omega}}, \qquad (13)$$

$$\bar{\varepsilon} = \frac{\exp[-2\exp(-\alpha/\Omega)\,\Phi\,\bar{t}/\Omega]}{\left\{1 + \dfrac{1-\exp[-2(\exp(-\alpha/\Omega)-\exp(-1/\Omega))\,\Phi\bar{t}/\Omega]}{2(\exp(-\alpha/\Omega)-\exp(-1/\Omega))\,\Phi/\Omega}\right\}^2}. \qquad (14)$$

Graphs of the change in turbulent energy and viscous dissipation, constructed by means of (13) and (14) for relatively moderate values of $\bar{t}$ (when the analysis performable is indeed valid) are presented in Fig. 1. The solution (12) for a single-phase medium is shown by dashes while the solid lines correspond to the values $\Phi = 1$ and $\alpha = 0.1$. It is seen that the presence of sufficiently inert particles ($\Omega \gtrsim 1$) results in more rapid degeneration of the turbulence while the presence of fine particles ($\Omega \ll 1$) causes a diminution in the damping rate of the fluctuation energy. The particles influence on viscous dissipation results in the opposite effects.

The influence of particle inertia on the ratio between the turbulent energy for $\Phi = 1$ and the corresponding quantity for $\Phi = 0$ for $\bar{t} = 1$ is shown in Fig. 2. It is clear to see that as Ω grows the turbulizing influence of the particles is replaced by a laminatizing effect. As the parameter α increases, the laminatizing action of the particles is expressed all the more clearly while the turbulizing effect is lowered for small Ω. In the limit cases $\Omega \to 0$ (with the exception of $\alpha = 0$) and $\Omega \to \infty$) the particle influence on turbulence vanishes.

The lowering of the turbulent energy upon the introduction of particles into the flow is explained by the additional dissipation because of the interphasal fluctuation slip described by the second term in the right side of (10). The maximam influence on $\bar{e}$ is observed for $\Omega \approx 1$, hwere, as is seen from Fig. 2, which corresponds to the maximum of the expression $\exp(-1/\Omega)/\Omega$. The turbulizing action of fine particles is explained by the fact that the maximum of their action on $\bar{\varepsilon}$ is found for $\Omega = \alpha$, as follows from (11), consequently, a diminution in the viscous dissipation $\bar{\varepsilon}$ results in a stronger effect of the influence of particles on $\bar{e}$ than the appearance of the additional interphasal dissipation. From the physical viewpoint, this is related to the circumstance that fine particles cause, without interacting with energetic fluctuation, suppression of the high-frequency part of the spectrum responsible for turbulent energy dissipation. Therefore, on the whole the nature of particle influence on degeneration of homogeneous turbulence turns out to be the same as in the near-wall flows although the turbulization upon the introduction of fine particles is not due to additional generation of turbulence but to diminution of viscous dissipation because of interphasal fluctuation slip.

NOTATION

u_i, U_i, v_i, V_i are the actual and averaged gas and solid phase velocities; ρ_1, ρ_2 are the gas and particle densities; P is the pressure; ν is the kinematic viscosity coefficient; φ, $\Phi = \rho_2\varphi/\rho_1$ is the volume and weight concentrations of the solid phase $\tau = 2\rho_2 r^2/9\rho_1\nu$ is the relaxation time of particles of radius r; $e = \langle u_k' U_k' \rangle/2$ is the turbulent energy; $Re = e^2/\varepsilon\nu$ is the Reynolds number; $\bar{t} = t\varepsilon_0/e_0$; $\bar{e} = e/e_0$; $\varepsilon = \bar{\varepsilon}/\varepsilon_0$. The subscript 0 denotes initial time.

LITERATURE CITED

1. Yu. A. Buevich and Yu. P. Gupalo, Prikl. Mekh. Tekh. Fiz., No. 4, 89-96 (1965).
2. A. A. Shraiber, L. B. Gavin, V. A. Naumov, and V. P. Yatsenko, Turbulent Gas Suspension Flows [in Russian], Kiev (1987).
3. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Inzh.- Fiz. Zh., 45, No. 4, 554-560 (1983).
4. I. V. Derevich, V. M. Eroshenko, and L. I. Zaichik, Inzh.-Fiz. Zh., 53, No. 5, 740-751 (1987).
5. G. Batchelor, Theory of Homogeneous Turbulence [Russian translation], Moscow (1955).
6. V. M. Ievlev, Turbulent Motion of High-Temperature Continuous Media [in Russian], Moscow (1975).

CONCENTRATION FIELD OF PARTICLES EJECTED INTO A NONHOMOGENEOUS ATMOSPHERE BY A MOVING SOURCE

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We obtain and analyze analytical expressions describing the space-time evolution of the density of solid particles ejected into the atmosphere.

Solid or liquid particles ejected as combustion products from aircraft engines are one of the important factors of man's influence on the earth's atmosphere [1]. In solid-fueled engines up to 1/3 of the weight of the exhaust is made up of metal-oxide particles whose typical size is of the orders of a few microns [2, 3]. The purpose of the present paper is to study theoretically the space-time structure of the exhaust trail of particles produced by a moving aircraft.

In the dense layers of the atmosphere the particles are carried along completely by the gaseous components of the exhaust and the distribution of particles is therefore similar to the concentration distributions of the gas components. However it is known that the velocity relaxation length (i.e., the distance the particle falls during the velocity relaxation time) in the important case of large knudsen number is inversely proportional to the density of the medium [4, 5], whereas the transverse size of the exhaust, gas jet is inversely proportional to the square root of the density [6]. Therefore at a certain altitude of flight the size of the cloud of particles begins to exceed the transverse dimensions of the region occupied by the gas components of the exhaust. In the first approximation, for altitudes greater than this critical height one can assume that the distribution of particles in the exhaust trail is formed as a result of ejection of particles into a nonmoving atmosphere from a moving point source, which corresponds to the near-nozzle flow region determining the initial velocity of the particles.

Then the equation of motion for the distribution function f of particles of a given size is:

$$\frac{\partial f}{\partial t} + \mathbf{v}\frac{\partial f}{\partial \mathbf{x}} + \frac{\partial}{\partial \mathbf{v}}((\mathbf{g}+\mathbf{a})f) = 0. \tag{1}$$

The drag force on a particle in the medium is assumed to be proportional to its velocity:

$$\mathbf{a} = -\gamma\cdot\mathbf{v}, \tag{2}$$

where the dependence of the reciprocal of the relaxation time on height is approximated as an exponential:

$$\gamma = \gamma(x_3) = \gamma(0)\exp(-x_3H^{-1}), \tag{3}$$

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